Enhancing Seismic Resilience of Water Pipe Networks *

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Abstract

As disasters such as earthquakes and floods become more frequent and detrimental, it is increasingly important that water infrastructure resilience be strategically enhanced to support post-disaster functionality and recovery. In this paper, we focus on the problem of strategically building seismic-resilient pipe networks to ensure direct water supply to critical customers and certain proximity to water sources for residential areas, which we formalize as the Steiner network problem with coverage constraints. We present an efficient mixed-integer linear program encoding to solve the problem. We also investigate the problem of planning partial network installments to maximize efficiency over time and propose an effective sequential planning algorithm to solve it. We evaluate our algorithms on synthetic water networks and apply them to a case study on a water service zone in Los Angeles, which demonstrate the effectiveness of our methods for large-scale real-world applications.

1 Introduction

Earthquakes can pose serious threats to water supply by damaging underground water pipes It is vital to have informed, effective, and efficient disaster mitigation and planning capabilities since water infrastructure is critical as it provides access to drinking water, functioning fire departments, and healthcare services. Hence, designing strategies to fortify water infrastructure in order to improve resilience to earthquakes is an important problem relevant to many cities across the USA and the globe. Los Angeles is one of the cities at the forefront of proactively prioritizing infrastructure resilience. In particular, fortifying the Los Angeles water pipe network to prepare for an earthquake is one of the primary actions the city is taking within the "Improved Infrastructure" goal to make a more resilient Los Angeles [Garcetti, 2018]. Placing seismically robust pipes at key locations helps improve continuous water delivery service and reduce the workload to restore water in areas suffering from water shortage after an earthquake. The City of Los Angeles and LADWP are planning to form a system-wide Seismic-Resilient Pipe Network (SRPN) to ensure sustained water delivery at critical locations. In particular, delivering water to critical customers responsible for public health and safety as well as customers providing community resilience services is of utmost importance [Davis, 2017]. Earthquake risk zones also cover large residential and commercial areas that are relatively isolated from the critical customers targeted by the SRPN. To guarantee service restoration rates, it is also important to ensure that all residents will be within a certain proximity to running water via SRPN regardless of the primary piping system's viability.

In this paper, we develop optimization approaches to strategically identify where old pipes should be replaced by seismic resilient pipes, respecting connectivity constraints on critical customers and coverage constraints on residential areas. To address the challenges in planning the SRPN, we propose the edge-weighted Steiner Network Problem with Coverage Constraints (SNP-CC). To solve SNP-CC, we present an efficient flow-based mixed-integer linear program (MILP) encoding of the problem with only a linear number of variables and constraints.

Even if we can obtain the optimal plan for the SRPN, it is not practical to complete the installation of earthquake-resistant pipes in a short period due to the limited budget and resources, the large scale of planned areas, and the difficulty of on-site constructions. Since Los Angeles started fortifying the water network with earthquake-resistant pipes, a total of 13,600 feet (2.58 miles) of them have been installed so far and they plan to upgrade about 14 miles of pipes in a three-year period starting in 2018 [LADWP, 2017], in contrast to the total length of 7,000 miles of pipe. In this situation, it becomes an important task to effectively determine which part of the optimal plan should be selected for each period of installment. We introduce an optimization problem to plan for optimal partial installments with the objective of satisfying as many of the water provision requirements as soon as possible over the planning time horizon within sequential budget allotments. We show that this problem is strongly NP-Hard to solve and propose a dynamic programming-based sequential planning algorithm.

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In experiments, we test our proposed approaches on synthetic water network graphs and a large-scale real-world water service zone. We compare the performance with a myopic baseline and demonstrate that our algorithms are able to handle instances with almost 8,000 threatened pipes and provide high-quality water pipe network designs as well as efficient installment plans.

1.1 Related Work

Research related to seismic resilience of water infrastructure includes designing simulation frameworks for risk and resilience assessments [Yoon *et al.*, 2018; Klise *et al.*, 2017; Tabucchi *et al.*, 2010] and optimizing post-earthquake water service recovery [Bellagamba *et al.*, 2019; Memarzadeh and Pozzi, 2019]. Our work is closely related to study in disaster response and pre-disaster planning, in which the goal is to optimize road network resilience and accessibility [Wu *et al.*, 2016; Yücel *et al.*, 2018; Gupta and Dilkina, 2019; Gupta *et al.*, 2018], and facilitate emergency evacuation [Kumar *et al.*, 2016; Romanski and Van Hentenryck, 2016; Even *et al.*, 2015].

The group Steiner tree problem is related to our network design problem. In that problem, at least one node from each given group of Steiner nodes should be included in the tree; while our design objective is a general network, we have constraints on subsets of edges as well as Steiner nodes. Theoretical results of hardness as well as approximation algorithms on group Steiner tree problem have been established [Demaine *et al.*, 2009; Charikar *et al.*, 1998; Garg *et al.*, 2000] . However, no practical tool or technique has been developed that we can borrow to solve our problem in large scales and meet real-world optimality requirements.

2 Steiner Network with Coverage Constraints

To capture the SRPN planning problem as a network design problem, we model the water network as an undirected graph $G = (V, E)$, where pipes in the water network are edges and the intersections of pipes are nodes. Each pipe e is associated with a non-negative replacement cost $c(e) \geq 0$. The set of water sources T is a subset of V. Let C be the set of critical customers. Given the location of each critical customer, the customer is assigned to the closest node in the network from which it can extract water; therefore, we can assume $C \subseteq V$. Denote by R the set of housing areas. For each $r \in R$, let $S(r) \subseteq E$ be the set of pipes that are close enough to serve r.

Formally, we define SNP-CC as follow:

Given: an undirected graph $G = (V, E)$, the sets of Steiner nodes C, sources T, coverage constraints R and $\{S(r) : r \in R\}$, the cost function on the edges $c : E \to R^{\geq 0}$.

Find: a set of edges $E' \subseteq E$ with minimum cost $\sum_{e \in E'} c(e)$, such that all $u \in C$ and $r \in R$ are satisfied. We say that a Steiner node $u \in C$ is *satisfied* if in the induced subgraph of E', u is connected to a source in T and r is *satisfied* if there exists $e \in E' \cap S(r)$ connected to a source in T.

W.l.o.g., we assume $T \cap C = \emptyset$ and $T \cap \{u : \exists(u, v) \in S(r)\} = \emptyset$, $\forall r \in R$. The problem is NP-Hard to solve and cannot be approximated to a factor of $o(\ln |R|)$ in polynomial time. Next, we present an efficient flow-based formulation that requires only $O(|V| + |E|)$ variables with $O(|V| + |E| + |R|)$ constraints.

We first transform the undirected graph G to a directed graph $\hat{G} = (V \cup \{0\}, \hat{E} \cup \{(0, t) : t \in T\})$ by replacing each undirected edge in E with two directed edges (collected in set $\hat{E}$) and adding a super source 0 that connects to all $t \in T$. Let $\delta^+(v)$ and $\delta^-(v)$ denote the sets of outgoing and incoming edges of vertex v in the transformed graph $\hat{G}$. Our overall Flow-Based MILP (FB-MILP) encoding of the problem is:

FB-MILP :

$$\min_{x,y,z} \sum_{(i,j)\in E} c(i,j)(x_{i,j} + x_{j,i})$$

$$\sum_{e\in\delta^-(v)} y_e = \mathbf{1}_{[v\in C]} + \sum_{e\in\delta^+(v)} (y_e + x_e) \qquad \forall v \in V \quad (1)$$

$$x_{i,j} + x_{j,i} \leq 1 \qquad \forall (i,j) \in E \quad (2)$$

$$\sum_{(i,j)\in S(r)} x_{i,j} + x_{j,i} \geq 1 \qquad \forall r \in R \quad (3)$$

$$0 \leq y_e \leq (|\hat{E}| + |V|)x_e \qquad \forall e \in \hat{E} \quad (4)$$

$$z + \sum_{t\in T} y_{0,t} = |\hat{E}| + |V| \quad (5)$$

$$\sum_{t\in T} y_{0,t} = |C| + \sum_{e\in\hat{E}} x_e \quad (6)$$

$$x_e \in \{0, 1\} \qquad \forall e \in \hat{E} \quad (7)$$

For each $e \in \hat{E}$, we introduce a binary variable x_e representing whether e is selected and a non-negative continuous variable y_e representing the number of units of flow on edge e. We take the nodes in T as the source of the flow network and span the graph by sending flow out of nodes in T. We let each edge e consume 1 unit of flow if e is used in the solution, i.e., $y_e > 0 \Rightarrow x_e = 1$, which is ensured by Constraint (4). Constraint (1) models the flow conservation. In Constraint (1), a node will absorb 1 unit of flow if v is a Steiner node. Constraint (3) makes sure at least one edge from $S(r)$ is selected. We give the system a total of $|\hat{E}| + |V|$ units of flow.Constraint (5) states that the residual flow z plus the total flow injected to the sources corresponds to the total flow, where $y_{0,t}$ ($t \in T$) represents flow injected to source t. Constraint (6) enforces that the flow consumed by the system corresponds to the total flow injected to sources.

3 Optimal Partial Network Installments

Given the optimal plan for the SRPN, a realistic approach for the water department to construct the SRPN is to split the plan into several installments, each given a certain amount of budget. Therefore, the key question raised given this setting is what pipes should be selected from the given global plan for each installment to maximize efficiency over time?

Suppose that given an instance of SNP-CC, we have already obtained the optimal solution represented by a set of edges E_{OPT}. We extend the cost function on edges to a set function for any subsets of E, i.e., let $c(E') =$

$\sum_{e \in E'} c(e) \, \forall E' \subseteq E$. Denote by $B = c(E_{\mathsf{OPT}})$ the total cost of the plan. We want to split the total cost B across n installments. Suppose the time horizon is $[0, 1]$ and the i-th installment is planned to be done at time $\frac{i-1}{n}$ with a budget of B_i allocated. Formally, we describe our problem as follow:

Given: an instance of SNP-CC and its optimal solution E_{OPT}, n time steps and $(B_1, \ldots, B_n)$ the budget allocation of the plan where $\sum_{i=1}^{n} B_i = B = c(E_{\mathsf{OPT}})$, $U : 2^{E_{\mathsf{OPT}}} \to R^{\geq 0}$ an utility function that evaluates the efficiency of any partial plan.

Find: the installment plan $(E_1, \ldots, E_n)$ such that $\cup_{i \leq n} E_i = E_{\mathsf{OPT}}$, $c(\cup_{j \leq i} E_j) \leq \sum_{j \leq i} B_i \, \forall i \leq n$, that maximizes the accumulated efficiency over time $\mathsf{EFF} = \frac{1}{n} \sum_{i=1}^{n} U(\cup_{j \leq i} E_j)$.

For an expository purpose, we define $U(E')$ to be the number of Steiner nodes satisfied by E' in the induce subgraph of E_{OPT} in the rest of this paper. This definition of $U(\cdot)$ corresponds to the number of satisfied critical customers in the SRPN. However, we will see that our algorithm could accommodate a variety of utility functions, including any *non-negative additive set function* $U(\cdot)$, that quantifies efficiency in practice, e.g., the number of leaky pipes fixed by the plan.

Next, we show the hardness of solving this problem in the following theorem.

Theorem 1. *Finding the optimal installment plan is strongly NP-Hard.*

To circumvent the computational hardness, we provide a sequential planning algorithm (SeqPlan) that works as follow: (i) initialize $E' = \emptyset$ and $\mathsf{EFF} = 0$, and plan for time step $1, \ldots, n$ sequentially; (ii) given the current plan $E' = \cup_{j<i} E_j$ at time step i, we greedily choose E_i that leads to the largest increment in utility, i.e., we define

$$\mathsf{PartialPlan}(E', B') = \arg \max_{E'_i : c(E'_i \cup E') \leq B'} U(E'_i \cup E')$$

and let $E_i = \mathsf{PartialPlan}(E', \sum_{j \leq i} B_i)$; (iii) add E_i to the current plan and update EFF; (iv) if $i < n$, continue to (ii) with the next time step, otherwise return the plan and EFF.

Proposition 1. SeqPlan *is optimal when* $n = 2$.

Next, we give the definition of non-overlapping coverage constraints and explore its property in Proposition 2.

Definition 1. *(Non-overlapping coverage constraints) For any* $r \in R$*, let* $V(r) = \{u : \exists (u, v) \in S(r)\}$ *be the set of nodes in the induced subgraph of* $S(r)$*. We have non-overlapping coverage constraints if* $V(r_i) \cap V(r_j) = \emptyset \, \forall r_i, r_j \in R$.

Proposition 2. *Assuming non-overlapping coverage constraints, there exists an optimal solution to* SNP-CC *that forms a forest, i.e., the induced graph of* E_{OPT} *is a forest. Each component in the forest contains exactly one node in* T.

In the rest of this section, we focus on the case assuming non-overlapping coverage constraints. For the utility function, since E_{OPT} forms a forest, we can let $U(e) = 1$ if the child node of e is a Steiner node and $U(e) = 0$ if not. To find $\mathsf{PartialPlan}(E', B')$, we can contract all components connected by E' in the induced graph of E_{OPT}, then transform the contracted graph into a rooted tree, and use dynamic programming (DP) to find the optimal solution.

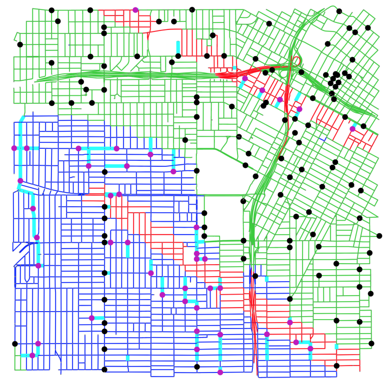


Figure 1: A visualized solution to one of the 4 miles×4 miles instances using roads as surrogates for water pipes. Pipes chosen by our algorithms are highlighted using thick cyan lines. Red and blue pipes are threatened pipes crossing fault zones and within liquefaction areas, respectively. Thin green pipes are non-threatened pipes. Safe customers, who are connected to water sources via safe pipes and located near to a water source, are marked as black dots; threatened customers are marked as pink dots. Water sources are not displayed in the figures.

4 Experiment

In experiments, we test our algorithms on synthetic water networks utilizing open-source road data and conduct a case study on a water service zone in Los Angeles. We provide extensive results to demonstrate the effectiveness of our proposed methods.

We use a myopic planning algorithm (MyoPlan) as the baseline that, given a fixed budget B_0 at each time step, plans greedily by optimizing the number of newly-satisfied customers and housing areas using a budget-constraint version of the flow-based MILP (denoted as BC-MILP). MyoPlan terminates when all customers and housing areas are satisfied.

4.1 Data Description and Preprocessing

We apply our approaches on synthetic water networks and the Service Zone 1134 in Los Angeles, where we utilize three pieces of data: i) road networks as surrogates for water networks from OpenStreetMap [OpenStreetMap, 2020] for the synthetic setting and a GIS representation of the water network in Zone 1134, including connectivity information, geographical data and basic features of water pipes and joints; ii) geographical data of critical customers; iii) GIS representations of fault zones and liquefaction areas in Los Angeles. From the data, we are able to obtain the graph representation $G = (V, E)$ of the water network, the set of critical customers C, the set of housing areas R as well as $S(r) \, \forall r \in R$ and the cost function on E. We also identify threatened pipes as those that are within a liquefaction area or within 500 feet of a fault zone. We guarantee non-overlapping coverage constraints and 0.75-mile proximity to water supply for all nodes empirically in all solutions generated by baselines and our algorithms in experiments.

For Zone 1134, we provide statistics in Table 1a. The numbers of nodes and pipes are the size of V and E, respectively. Safe pipes are guaranteed connection to water sources even

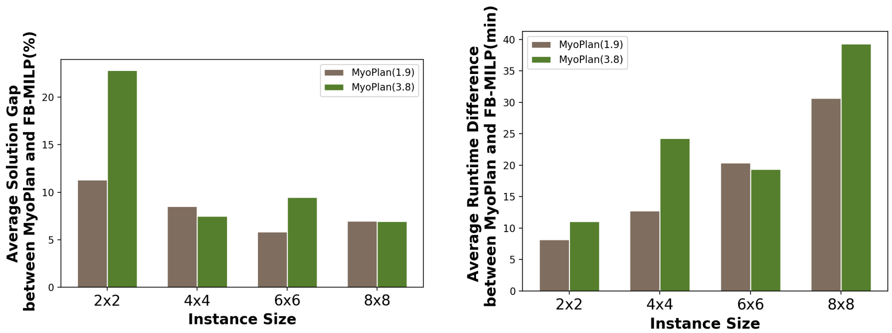
Pipes	34,462
Safe Pipes	25,609
Threatened Pipes	8,434
Nodes	31,674
Threatened Nodes	232
Critical Customers	298
Threatened Critical Customers	93

(a) Statistics about Zone 1134.

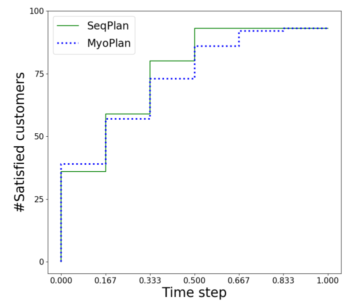
	Cost (Gap)	Time/min	EFF	
FB-MILP	**23.47 (0)**	**18.05**	MyoPlan	SeqPlan
MyoPlan(1.9)	24.37 (3.83%)	31.76	59.30	**59.50**
MyoPlan(2.5)	24.53(4.53%)	29.38	57.40	**58.30**
MyoPlan(3.2)	24.18 (3.03%)	23.35	57.38	**58.00**
MyoPlan(3.8)	24.22 (3.21%)	22.55	56.8	**57.71**
MyoPlan(4.8)	24.30 (3.53%)	20.88	57.80	**60.16**

(b) Solution quality of FB-MILP and MyoPlan, and plan efficiency of SeqPlan and MyoPlan.

Table 1: Statistics and results of Zone 1134.



(a) Solution gaps and runtime differences between FB-MILP and MyoPlan of each instance size. MyoPlan with budget B_0 is denoted by MyoPlan(B_0).



(b) The utility curve for $B_0 = 4.8$ miles.

Figure 2: Experimental results.

if all threatened pipes fail. Besides safe pipes and threatened pipes, there are 449 pipes connected to water sources via at least one threatened pipe and will be isolated from the water supply in the worst case. In addition, we count the number of non-covered nodes and threatened critical customers. Non-covered nodes are nodes not within 1 mile proximity to any safe pipe or water source. Threatened critical customers are those connected to water sources via at least one threatened pipe and will lose direct water supply in the worst case.

4.2 Results on Synthetic Water Pipe Networks

We run three sets of experiments on each of the three locations we picked on the map of Los Angeles. For each location, we use its coordinate as the center to crop the road networks from OpenStreetMap using $L \times L$ ($L = 2, 4, 6, 8$ miles) bounding boxes to approximate the water pipe network. For each L, we run FB-MILP and MyoPlan ($B_0 = 1.9, 3.8$ miles) and compare their runtime and solution quality. The cutoff time for FB-MILP to solve a $L \times L$ instance is set to $15L$ minutes. For MyoPlan, we first estimate the time step needed as $n' = \lceil c(E_{OPT})/B_0 \rceil$ and set the cutoff time to $18L/n'$ minutes for each solve of BC-MILP. We show the solution gap and runtime difference between MyoPlan and FB-MILP in Figure 2a. We can see that FB-MILP outperforms MyoPlan by 6% to 23% in solution quality and is above 30 minutes faster than MyoPlan on the largest instances. We visualize the solutions of FB-MILP to one of the 4 miles$\times$4 miles instances in Figure 1.

4.3 Case Study on Zone 1134

Given the instance to SNP-CC extracted from data, we run FB-MILP to compute the optimal solution. We also run MyoPlan with different budgets. In Table 1b, we compare the solution quality and runtime with MyoPlan. FB-MILP is able to find the optimal solution with cost $c(E_{\mathsf{OPT}}) = 23.47$ in 18.05 minutes. In MyoPlan, we set the cutoff time for each solve of BC-MILP to 150 seconds for $B_0 = 1.9, 2.5$ miles, 200 seconds for $B_0 = 3.2, 3.8$ miles, and 250 seconds for $B_0 = 4.8$ miles. We can see that MyoPlan with different budgets finds sub-optimal solutions that cost 3.03% to 4.53% higher than E_{OPT}.

Given the global optimal plan E_{OPT}, we investigate how we should plan for partial installments. We show the efficiency of SeqPlan and compare it with MyoPlan that plans without the guidance of the optimal solution. We set $B_0 = 1.9, 2.5, 3.2, 3.8, 4.8$ miles and show the solution quality in Table 1b. We allocate the same budget as MyoPlan at each time step for SeqPlan. The EFF of SeqPlan takes less than 2 seconds to compute in all instances. We can see that in all cases, SeqPlan dominates MyoPlan in plan efficiency. In Figure 2b, we show the utility curve for $B_0 = 4.8$ miles. In this case, MyoPlan achieves a higher utility for the first time step. However, after the second time step, MyoPlan is dominated by SeqPlan and takes two more time steps than SeqPlan to cover all customers.

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